A real space renormalisation group study of the Ising model on self-avoiding walk chains

1985 J. Phys. A: Math. Gen. 18 1037

(http://iopscience.iop.org/0305-4470/18/6/026)

J. Phys. A: Math. Gen. 18 (1985) 1037-1040. Printed in Great Britain

COMMENT

A real space renormalisation group study of the Ising model on self-avoiding walk chains

B K Chakrabarti†§ and S Bhattacharya‡

† Institute of Theoretical Physics, University of Cologne, Zulpicher Strasse 77, 5000 Cologne 41, West Germany

‡ Saha Institute of Nuclear Physics, 92 Acharya Prafulla Chandra Road, Calcutta 700009, India

Received 19 November 1984

Abstract. Here we apply a small cell, real space renormalisation group technique to study the critical behaviour of the nearest-neighbour interacting Ising model on self-avoiding walk (SAW) chains. In the limit of vanishing interactions between nearest neighbours which are not connected by the walk itself, the treatment reproduces exactly the critical behaviour of the linear chain Ising model. When all the nearest-neighbour interactions are allowed, the treatment gives a vanishing T_c, with the small cells chosen, for the Ising model on SAWS on the square and triangular lattices. The study of the critical behaviour of such models indicates some interesting departures from that of the Ising model on linear chains.

Recently we have proposed and studied the phase transition and critical properties of an Ising model with nearest-neighbour interactions on self-avoiding walk (SAW) chains, where the SAWS themselves are executed on any d-dimensional lattice (Chakrabarti and Bhattacharya 1983, Bhattacharya and Chakrabarti 1984a, b). One can apply such a lattice statistical model in the study of phase transitions in magnetic (linear) polymers, and also in connection with studies on the magnetism of disordered solids near the percolation threshold. Unlike spins on a linear chain, the spins on a SAW chain frequently have more than two nearest neighbours, which indicates a finite average transition temperature T_c, even with short range interactions, for such Ising models on SAW chains. The critical behaviour of such a system is also interesting, as the system feels all the d-dimensional fluctuations with a particular kind of quenched randomness (with excluded volume effects). The finiteness of the transition temperature of such a quasilinear system is also indicated by the fact that the lower critical dimensionality of the Ising model, with short range interactions, is unity (see e.g. Boccara and Havlin 1984) and that the fractal dimensionality D of such a system on SAWS is greater than one (and less than two for $d<4$) (see e.g. Mandelbrot 1982). Also the critical behaviour of such a system should correspond to the fractal dimensionality D of the system, rather than to the Euclidean dimension d of the lattice on which SAWS are executed (cf Gefen *et al* 1983).

Employing the Bethe-Peierls approximation $kT_c/J=-2/\ln(1-2/Z_{\text{eff}})$, where J is the nearest-neighbour exchange interaction and Z_{eff} is the average number of nearest

§ On leave from Saha Institute of Nuclear Physics, Calcutta 700009, India.

0305-4470/85/061037+04\$02.25

neighbours for spins on a SAW, and using $Z_{\rm eff} = 2+(Z-1)-\mu$, where Z is the lattice coordination number and μ is the average connective constant for SAWs on the lattice (see e.g. McKenzie 1976), we found $kT_c/J < 1.06$ and 1.65 for Ising models on SAWs on the square and triangular lattices respectively(Bhattacharya and Chakrabarti 1984a). Using, on the other hand, computer simulation results for the percolation thresholds (p_c) for site dilution on SAW chain lattices, and employing the semi-empirical formula (Bishop 1973) $kT_c = Z_{\rm eff}[(p_c^{-1}+1)\tanh^{-1} p_c]^{-1}$, connecting p_c's with the Ising model T_c's on the same lattices, we found $kT_c/J > 0.51$ and 0.78 for Ising models on SAWs on the square and triangular lattices respectively (Bhattacharya and Chakrabarti 1984a). Studying similar Ising models on quasilinear fractals, Gefen *et al* (1983) showed that the correlation length exponent ν_T for the Ising models on such fractals is given by $\nu_T = D^{-1}$ ($=0.75$ for such Ising models on SAWs on $d=2$ lattices). Using and extrapolating the computer simulation results for the shortest connecting path lengths in SAWs on both the square and triangular lattices, and integrating the spin correlations on them, we found the susceptibility exponent $\gamma_T = 1.024 \pm 0.007$ for such Ising models on SAWs on two-dimensional lattices (Bhattacharya and Chakrabarti 1984a). The critical dynamics of Heisenberg spins on SAW chains was studied using a scaling picture and applying a real space renormalisation group (RSRG) technique (Bhattacharya and Chakrabarti 1984b), and the study indicated $z = 2D/\gamma_T$ for the value of the dynamical exponent z.

Here, we apply a small cell RSRG technique to such Ising models on SAWs on the square and triangular lattices. In the limit of vanishing interactions between the nearest neighbours which are not connected by the walk itself, such a treatment gives, as one can easily check, the exact linear chain critical behaviour for the Ising model. (It may be mentioned that the application of the replica trick and momentum space renormalisation group technique does not reproduce this result in this trivial limit (Chakrabarti and Bhattacharya 1983).) When all the nearest-neighbour interactions are allowed, our treatment gives vanishing T_c's. Also the study of critical behaviour indicates an interesting change from that of the Ising model on linear chains.

Let us choose the cells shown in figures 1(a) and 1(b) for the square and triangular lattices respectively. The recursion relations for the renormalised fugacities f' may then be written as (cf Stanley *et al* 1982, Shapiro 1978)

$$f' = f^2 + 2f^3 + f^4, \tag{1}$$

$$f' = 2f^2 + 2f^3, \tag{2}$$

for SAWs on the square and triangular lattices respectively. If now the Ising exchange interactions J are allowed only along the walk, then, with $t = \tanh(J/kT)$, the renor-

(a) (b)

Figure 1. Original and renormalised cells are shown for (a) the square lattice (scale factor $b=2$) and (b) the triangular lattice ($b=\sqrt{3}$).

malised interactions t' are trivially given by

$$t'f' = t^2f^2 + 2t^3f^3 + t^4f^4, \tag{3}$$

$$t'f' = 2t^2f^2 + 2t^3f^3, \tag{4}$$

for the Ising models on SAWs on the square and triangular lattices respectively. If the interactions are allowed between any occupied nearest neighbours, the renormalised interactions are then given by (cf Yeomans and Stinchcombe 1980)

$$t'f' = t^2f^2 + 2t^3f^3 + \left(\frac{t^2+t^4}{1+t^4}\right)f^4, \tag{5}$$

$$t'f' = 2t^2f^2 + 2\left(\frac{2(t^2+t^3)}{1+2t^3+t^4}\right)f^3, \tag{6}$$

for the Ising models on SAWs on the square and triangular lattices respectively.

The non-trivial fixed points (f^* and t^*) and the exponents (given by $(\mathrm{d}f'/\mathrm{d}f)_{(f^*,t^*)} = b^{1/\nu_{\mathrm{SAW}}}$ and $(\mathrm{d}t'/\mathrm{d}t)_{(f^*,t^*)} = b^{1-\nu_{\mathrm{SAW}}\nu_{\mathrm{T}}}$, where $b=2$ and $\sqrt{3}$ for the cells used in figures 1(a) and 1(b) respectively) are given in table 1, where these values are also compared with other exact results and best estimates. In the case of nearest-neighbour Ising interactions, the critical behaviour, however, shows, with both the cells used, interesting crossover to a new critical behaviour, different from that of the linear chain.

Table 1. Non-trivial fixed points and exponents for the recursion relations (1)–(6).

System		SAWs on							
		Square lattice				Triangular lattice			
		f^*	t^*	ν_{SAW}	ν_{T}	f^*	t^*	ν_{SAW}	ν_{T}
Nearest-neighbour interactions along the walk only (effectively 1D problem)	Equations (1) and (3) (2) and (4)	0.47	1	0.72	1	0.37	1	0.67	1
	Best estimates and exact results	0.38[a]	1	0.75[b]	1	0.24[a]	1	0.75[b]	1
All nearest-neighbour interactions allowed	Equations (1) and (5) (2) and (6)	0.47	1	0.72	1.14	0.37	1	0.67	2.16
	Best estimates	0.38[a]	0.74 to 0.96[c]	0.75[b]	—	0.24[a]	0.54 to 0.85[c]	0.75[b]	—

[a] McKenzie (1976).
[b] Nienhuis (1982).
[c] Bhattacharya and Chakrabarti (1984a).

Acknowledgment

We are grateful to D Stauffer for some useful criticisms and comments.

References

Bhattacharya S and Chakrabarti B K 1984a *Z. Phys.* B **57** 151
—— 1984b *J. Phys. A: Math. Gen.* **17** L761
Bishop A 1973 *J. Phys. C: Solid State Phys.* **6** 2089
Boccara N and Havlin S 1984 *J. Phys. A: Math. Gen.* **17** L547
Chakrabarti B K and Bhattacharya S 1983 *J. Phys. C: Solid State Phys.* **16** L1025
Gefen Y, Aharony A and Mandelbrot B 1983 *J. Phys. A: Math. Gen.* **16** 1267
Mandelbrot B 1982 *The Fractal Geometry of Nature* (San Francisco: Freeman) p 329
McKenzie D S 1976 *Phys. Rep.* **27** 35
Nienhuis B 1982 *Phys. Rev. Lett.* **49** 1062
Shapiro B 1978 *J. Phys. C: Solid State Phys.* **11** 2829
Stanley H E, Reynolds P J, Redner S and Family F 1982 in *Real Space Renormalization* ed T W Burkhardt and J M J van Leeuwen (Berlin: Springer) p 190
Yeomans J M and Stinchcombe R B 1980 *J. Phys. C: Solid State Phys.* **13** 85